sary to determine χ from (4) and substitute its value in (16). If λ_0/D is greater than the right-hand side of (16), side coupling is preferable; if λ_0/D is less, then end coupling will give larger gaps.

Finally, in a practical design it is usually necessary that

$$\frac{\lambda_0'}{2} > D \tag{19}$$

in order to prevent the generation of TM modes [7] and so minimize loss by radiation. By rearranging (19) and replacing λ_0' by $\lambda_0/\sqrt{\epsilon_r}$, one imposes a further constraint on the permissible range of the λ_0/D ratio for end-coupled filters and for all values of characteristic impedance:

$$\frac{\lambda_0}{D} > 2\sqrt{\epsilon_r}. \tag{20}$$

To facilitate selection of the filter type, a graph has been prepared with λ_0'/D as a function of $\sqrt{\epsilon_r}\, Z_0$ (Fig. 5), incorporating (14), (16), and (20).

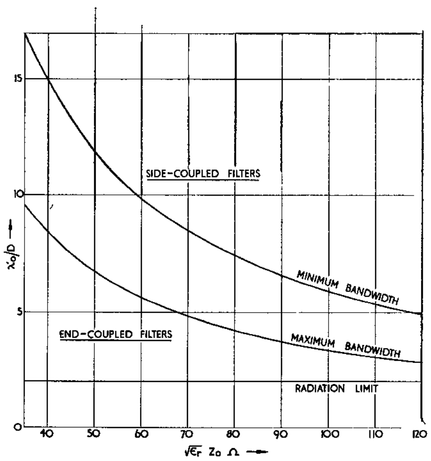

Fig. 5. λ_0'/D as a function of $\sqrt{\epsilon_r}Z_0$ for complete bandwidth range.

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An Interesting Impedance Matching Network

Any load impedance can be transformed to a real impedance by a $\lambda/8$ transformer whose characteristic impedance is equal to the magnitude of the load impedance.

The locus of a normalized load impedance, as a function of the characteristic impedance of the feed line, is a constant (X/R) contour on the Smith chart. Further, a constant (X/R) contour is a circular segment drawn through the points zero and infinity. The point of closest approach of any constant (X/R) contour to the $(Z/Z_0)=1$ point, minimum $|\Gamma|$, takes place along a straight line drawn from $(Z/Z_0)=j$ to $(Z/Z_0)=-j$, or $\lambda/8$ away from the real axis. Therefore, if any load is fed by a line whose characteristic impedance minimizes the magnitude of the reflection coefficient at the load, the normalized impedance $\lambda/8$ away from the load will be pure real.

Consider the matching network in Fig. 1.

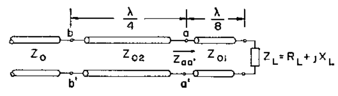

Fig. 1. Load-line impedance match.

The squared magnitude of the reflection coefficient at the load is given by

$$|\Gamma|^2 = \frac{|Z_L|^2 - 2R_LZ_{01} + Z_{01}^2}{|Z_L|^2 + 2R_LZ_{01} + Z_{01}^2} \tag{1}$$

where

$$R_L = \operatorname{Re}\{Z_L\}.$$

We may write

$$|\Gamma|^2 = 1 - \delta \tag{2}$$

where

$$\delta = \frac{4R_LZ_{01}}{|Z_L|^2 + 2R_LZ_{01} + Z_{01}^2}\cdot \tag{3}$$

Then

$$\frac{\partial\delta}{\partial Z_{01}} = 0 \tag{4}$$

implies that

$$(|Z_L|^2 + 2R_LZ_{01} + Z_{01}^2)(4R_L) - (4R_LZ_{01})(2R_L + 2Z_{01}) = 0 \tag{5}$$

or

$$Z_{01}^2 = |Z_L|^2. \tag{6}$$

Therefore, $|\Gamma|^2$ is a minimum if $Z_{01} = |Z_L|$. Since $|\Gamma| < 1$, $|\Gamma|$ will also be minimized by the same value of Z_{01}.

The driving point impedance of the $\lambda/8$ transformer, $Z_{aa'}$ in Fig. 1 is

$$Z_{aa'} = \frac{R_L}{1 - \dfrac{X_L}{|Z_L|}} \tag{7}$$

when

$$Z_{01} = |Z_L|.$$

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Thus if the Q of the load impedance is high,

$$Z_{aa'} \cong \begin{cases} \dfrac{R_L}{2}, & X_L < 0 \\ 2X_LQ_L, & X_L > 0 \end{cases} \tag{8}$$

where

$$Q_L = \frac{X_L}{R_L}\cdot \tag{9}$$

The impedance of the quarter-wave transformer in Fig. 1 may then be found from

$$Z_{01} = \sqrt{Z_{aa'}Z_0} \tag{10}$$

as is well known.

By application of the same principles, a conjugate match between two impedances Z_G and Z_1 may be accomplished in a $\lambda/2$ length of line as shown in Fig. 2, where

$$Z_{01} = |Z_L| \tag{11}$$

$$Z_{02} = \left[\frac{R_LR_G|Z_L||Z_G|}{(|Z_L| - X_L)(|Z_G| - X_G)}\right]^{1/2} \tag{12}$$

$$Z_{03} = |Z_G| \tag{13}$$

in which

$$Z_L = R_L + jX_L \tag{14}$$

$$Z_G = R_G + jX_G. \tag{15}$$

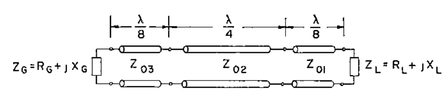

Fig. 2. Source-load impedance match.

These circuits do not constitute minimum length solutions to the impedance matching problem. They do provide a simple answer to the synthesis problem in cases where frequency response is not the primary concern, as is the case in many frequency multiplier design problems. In addition, the line lengths involved are always known in advance. This property has been found to be particularly useful in microwave circuit design when the exact value of the input impedance is not a priori known.

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A Laminar Slow-Wave Coupler and Its Application to Indium Antimonide

The purpose of this correspondence is to describe a technique for coupling energy between a waveguide mode and a semiconductor (or gas discharge) slow wave that has a longitudinal component of microwave electric

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